T-Shirt Explanation: Westmont's 31st Annual Mathematics Contest

Prime numbers of the form $2^n - 1$, where *n* is a positive integer, are called *Mersenne primes* in honor of Marin Mersenne (1588–1648), a French mathematician, theologian, philosopher, and music theorist. The number 5 is prime, but there is no integer *n* for which $2^n - 1 = 5$. Thus, not all primes are Mersenne primes. Likewise, since $2^4 - 1 = 15$, not all numbers of the form $2^n - 1$ are prime.

The T-shirt for Westmont's 31st annual mathematics contest, shown below, contains a design listing all the prime numbers up to 127. Among them only the numbers $3 = 2^2 - 1$, $7 = 2^3 - 1$, $31 = 2^5 - 1$, and $127 = 2^7 - 1$ are Mersenne primes, which is why they are printed more prominently on the T-shirt. That 31 is a Mersenne prime ties nicely with the fact that 2019 marked Westmont's 31st annual mathematics contest.



As of February 2019 there were only 51 known Mersenne primes. The 51st one was discovered in December, 2018, by a computer belonging to Patrick Laroche of Ocala, Florida. At some point he linked his computer to a network known as the *Great Internet Mersenne Prime Search*, or GIMPS for short. The network was formed in 1996, and anyone with a computer can join in the hunt for more Mersenne primes. For details see https://www.mersenne.org/.

The Mersenne prime found by Ocala's computer equals $2^{82,589,993} - 1$, which is an incredibly large number having 24,862,048 digits. Just to write it down using 12-point font would take up the space of about 12 novels the size of Anna Karenina!

There is a curious connection between Mersenne primes and *perfect numbers*, which have the property that they equal the sum of their proper divisors. Thus, 28 is perfect because its proper divisors are 1, 2, 4, 7, and 14, and 1 + 2 + 4 + 7 + 14 = 28. For an elucidation of this connection check out the T-shirt explanation for Westmont's 28th annual mathematics contest, which was held in 2016.

To this date no one knows whether there is an infinite supply of Mersenne primes. Would you like to earn instant fame for a mathematical discovery? All you need do is settle that issue!