22. Suppose that $G$ is a group with more than one element and $G$ has no proper, nontrivial subgroups. Prove that $|G|$ is prime. (Do not assume at the outset that $G$ is finite.)

Prove the contrapositive by supposing that $|G|$ is not prime. There are then 2 possible cases that result from this supposition.

Case 1: $|G| = \infty$.
Let $x \in G, x \neq e$. Either $|x| = \infty$ or $|x| = n$.
If $|x| = n$, then I have found an element in $G$ that generates a group of order less than the group. Thus, $G$ has a nontrivial proper subgroup.
If $|x| = \infty$, look at the group generated by $x^2$, $\langle x^2 \rangle$. It must be the case that $\langle x^2 \rangle < G$ since it does not contain $x$, which is in $G$.

Case 2: $|G| = pq$.
Let $x \in G, x \neq e$. Either $|x| = p$, $|x| = q$ or $|x| = pq$.
If $|x| = p$ or $|x| = q$, then $\langle x \rangle < G$ since $p < pq$ and $q < pq$.
If $|x| = pq$, look at $\langle x^p \rangle$. $\langle x^p \rangle = \{x^p, x^{2p}, x^{3p}, ..., x^{qp}\}$. There are $q$ elements, thus $|\langle x^p \rangle| = q$. I have found an element in $G$ that produces a nontrivial proper subgroup.

The contrapositive has been shown. Thus, if $G$ is a group with more than one element and $G$ has no proper, nontrivial subgroups, then $|G|$ is prime.

24. Let $G$ be a group of order 25. Prove that $G$ is cyclic or $g^5 = e$ for all $g$ in $G$.

Suppose that $G$ is not cyclic. Thus, there does not exist $g \in G$ that generates the group $G$. The orders of the elements in $G$ must divide 25 but cannot equal 25 because $G$ is not cyclic. Hence, $|g| = 1$ or 5. If $|g| = 1$, then $g = e$ and $g^5 = e^5 = e$. And if $|g| = 5$, then $g^5 = e$. And we can conclude that if $G$ is not cyclic, $g^5 = e$ for all $g \in G$. Therefore, if $G$ is a group of order 25, then $G$ is cyclic or $g^5 = e$ for all $g \in G$. 