Axioms for Mathematical Analysis

- Axiom: from the Greek word ἄξιος (axios), meaning worthy (see Revelation 4:11)

- Gottlob Frege
  - Founded the school of logicism, which attempted to equate mathematics with logic.
  - Produced *Grundgesetze der Arithmetik* (The Basic Laws of Arithmetic), a work that formalized his program.
  - While the *Grundgesetze* was in press, Frege received a letter from Bertrand Russell written on June 16, 1902, pointing out that Frege’s fifth axiom was inconsistent with the other axioms in the *Grundgesetze*.
  - Frege added an appendix to the Grundgesetze, commenting on Russell’s discovery.
  - Frege wrote to Russell on June 22, 1902, “With the loss of my Rule V, not only the foundations of my arithmetic, but also the sole possible foundations of arithmetic, seem to vanish.”

- Currently accepted axioms for set theory
  - ZFC = Zermelo-Fraenkel axioms, with the axiom of choice.
  - First given by Ernst Zermelo in 1908, then modified by Abraham Fraenkel in 1922.

- Axioms for the real numbers (Chapter 3). Note especially the following:
  - the well-ordering property;
  - the trichotomy law;
  - the completeness axiom; and
  - the Archimedian property (a theorem, not an axiom).

*Euclid Alone has looked on Beauty Bare*
(a poem by Edna St. Vincent Millay)

Euclid alone has looked on Beauty bare.
Let all who prate of Beauty hold their peace,
And lay them prone upon the earth and cease
To ponder on themselves, the while they stare
At nothing, intricately drawn nowhere
In shapes of shifting lineage; let geese
Gabble and hiss, but heroes seek release
From dusty bondage into luminous air.
O blinding hour, O holy, terrible day,
When first the shaft into his vision shone
Of light anatomized! Euclid alone
Has looked on Beauty bare. Fortunate they
Who, though once only and then but far away,
Have heard her massive sandal set on stone.