The Power of One: The Surprising Influence of an Individual

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Intro: Despite the common belief that the actions of a single individual do not make a difference within a large population, single individuals can have a surprisingly significant influence.

Model 1: Predator-Prey
Consider a 'Boid' simulation sheep herd
● move/graze based on nearby sheep
● run away when see a wolf
● a sentinel is a sheep with heightened sensitivity to predators
A small number of sentinels significantly improves the longevity of the herd.

Conclusion
These two models show empirically and analytically that a single individual can significantly improve collective outcomes. If people believe their actions make a difference, perhaps they will be more likely to do so.

Model 2: Voting
Consider a forced two-choice voting context
● we say outcome 'yes' is objectively good; the 'no' outcome objectively bad
● the population consists of two voter types:
  • naive cannot distinguish between the good and bad outcomes
  • naive voters choose a vote by flipping a fair coin
  • astute voters always select the good outcome

One astute voter exerts a disproportionate influence on the outcome relative to its representation
● as expected, the single astute voter tips the outcome slightly toward the good
● surprisingly, the relative influence of the astute voter increases with the population size
● although the absolute influence decreases, the influence relative to population size approaches infinity

\[
V(n) = \frac{1}{2^n} \sum_{i=\left\lfloor \frac{n}{2} \right\rfloor + 1}^{n} \binom{n}{i}
\]
probability 'good' wins among \( n \) naive voters

\[
V(a, n) = \frac{1}{2^n} \sum_{i=\left\lfloor \frac{n-a}{2} \right\rfloor + 1}^{n} \binom{n}{i}
\]
probability 'good' wins among a astute and \( n \) naive voters

\[
C(a, n) = V(a, n) - V(0, n)
\]
contribution of a astute voters among \( n \) naive voters

\[
C(a, n) = \frac{1}{2^n} \sum_{i=\left\lfloor \frac{n-a}{2} \right\rfloor + 1}^{n} \binom{n}{i}
\]